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earlier teaching in optics and the trigononometrical trace of a ray, undertook the line test.

The actual radii and the thickness of the triple constructed are given in the Appendix, and also a worked-out example of the foci (thin lens) by a short formula which has been condensed from Coddington by Mrs. Gifford.

Before closing this paper, I should like to thank Sir David Gill (to whom my bringing this forward is entirely due) for his great kindness; Messrs. Chance Bros., Messrs. Schott of Jena, and Messrs. Mantois of Paris, for the facilities they have afforded me for measuring their glasses; and last, but not least, Mr. F. Twyman, of Messrs. Hilger Ltd., for the rare skill he has shown in accurately following the optical data given.

## APPENDIX.

### *Reciprocals of Foci for Apochromatic Triple.*

$\mu$ is baryta light flint. $\mu'$ is borosilicate flint. $\mu''$ is borosilicate crown.

*Differences of Proportional Spectra.*

| A′—D. | D—F. | F—G′. |
|---|---|---|
| 0·00168 | −0·00030 | 0·00805 |

*Refractive Indices.*

| B′.7066(He). | A.5607(Pb). | Φ.4678(Cd6). |
|---|---|---|
| $\mu_1 = 1{\cdot}569074$ | $\mu_2 = 1{\cdot}576104$ | $\mu_3 = 1{\cdot}584492$ |
| $\mu'_1 = 1{\cdot}569909$ | $\mu'_2 = 1{\cdot}577675$ | $\mu'_3 = 1{\cdot}586869$ |
| $\mu''_1 = 1{\cdot}497250$ | $\mu''_2 = 1{\cdot}502400$ | $\mu''_3 = 1{\cdot}508365$ |

$$A = \{(\mu_3 - \mu_2)(\mu'_2 - \mu'_1) - (\mu'_3 - \mu'_2)(\mu_2 - \mu_1)\}(\mu''_2 - 1) = 0{\cdot}0000002549117.$$

$$B = \{(\mu''_3 - \mu''_2)(\mu_2 - \mu_1) - (\mu_3 - \mu_2)(\mu''_2 - \mu''_1)\}(\mu'_2 - 1) = -0{\cdot}0000007303256.$$

$$C = \{(\mu'_3 - \mu'_2)(\mu''_2 - \mu''_1) - (\mu''_3 - \mu''_2)(\mu'_2 - \mu'_1)\}(\mu_2 - 1) = 0{\cdot}0000005904547.$$

$$F = 1.$$

$$\frac{1}{f} = \frac{C}{A+B+C} = 5{\cdot}132568. \qquad \frac{1}{f'} = \frac{B}{A+B+C} = -6{\cdot}348405.$$

$$\frac{1}{f''} = \frac{A}{A+B+C} = 2{\cdot}215837.$$

$$\frac{1}{f} + \frac{1}{f'} + \frac{1}{f''} = 1. \qquad f' = -0{\cdot}157519.$$

$$\text{Corresponding curvature sum} = \frac{\frac{1}{f'}}{\mu'_2 - 1} = -10{\cdot}989579.$$

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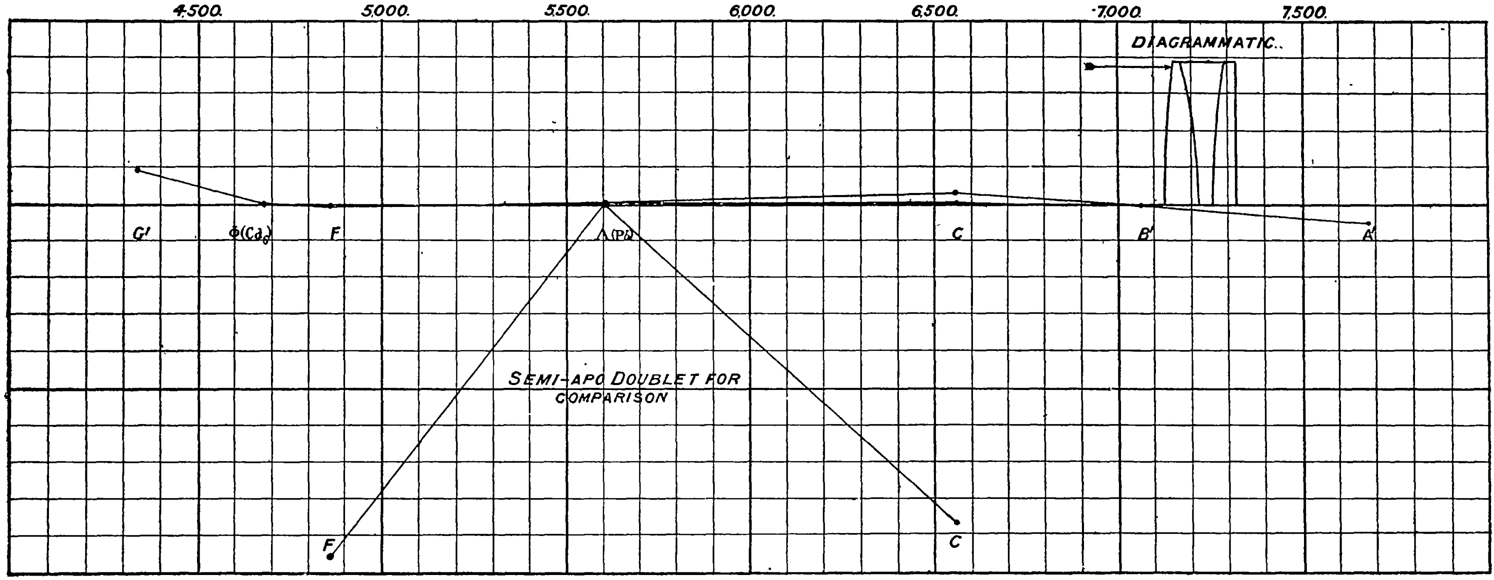
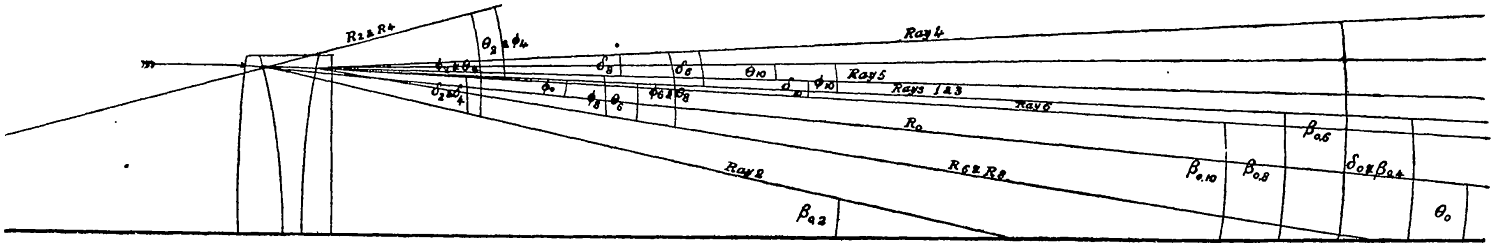
## CURVE OF FOCAL LENGTHS AND WAVE-LENGTHS.

For the purposes of the curve, the data given have been multiplied by twenty. For comparison, the doublet already referred to has been treated similarly. All the focal lengths are from trigonometrical traces for edge rays, not for axial rays.





TRIGONOMETRICAL TRACE.

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## *Optical Data by Trigonometrical Trace.*

Aperture, 3-inch. Focus, 37·5-inch. Ratio, 12·5.

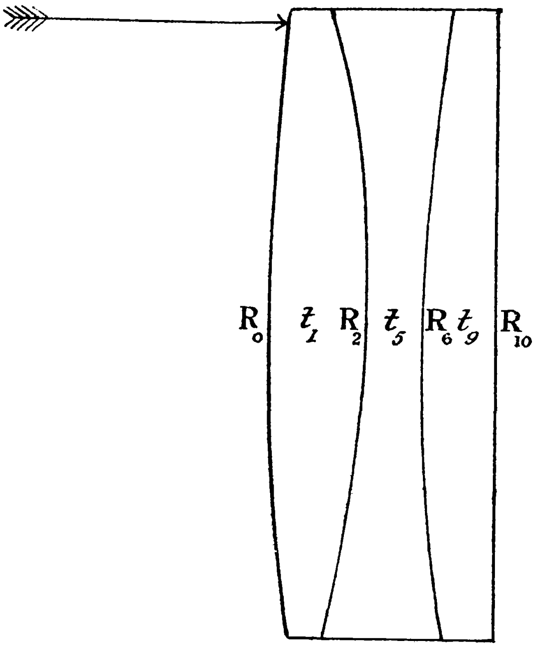
**Radii.**

$R_0 = 18{\cdot}28125$
$R_2 = -5{\cdot}54507$
$R_4 = R_2$
$R_6 = 9{\cdot}13636$
$R_8 = R_6$
$R_{10} = -138{\cdot}755$

**Thicknesses.**

$t_1 = 0{\cdot}450$
$t_3 = 0{\cdot}001$
$t_5 = 0{\cdot}150$
$t_7 = 0{\cdot}003$
$t_9 = 0{\cdot}350$

Diagrammatic.



*Note.*—In order to get rid of the last trace of spherical aberration it became necessary to make $R_4$ 0·001 shorter, and to alter $R_{6-8}$ and $R_{10}$ in proportion.

The following are the data calculated direct from the curvature sums without thickness, and starting from the same (arbitrary) radius $R_0$.

$R_0 = 18{\cdot}28125$
$R_2 = -5{\cdot}44237$
$R_4 = R_2$
$R_6 = 9{\cdot}09712$
$R_8 = R_6$
$R_{10} = -135{\cdot}180$

The influence of thickness, therefore, is to lengthen the radii.

It was thought that a few trigonometrical traces through another triple for ray Λ incident parallel to the axis at different heights from the vertex, and for other rays, would afford useful information with regard to the distance from the last surface along the axis at which each pencil intersects it, as follows:—

| Ray. | Height from Vertex. | * Back Focus. | Focus. |
|---|---|---|---|
| Λ | 0·00 | 28·98490 | 30·00000 |
| ,, | 0·45 | 28·98496 | 30·00109 |
| ,, | 0·90 | 28·98398 | 30·00425 |
| ,, | 1·20 | 28·98330 | 30·00751 |
| ,, | 1·35 | 28·98232 | 30·00906 |
| ,, | 1·50 | 28·98142 | 30·01084 |
| B′ | 0·00 | 28·99045 | 30·00140 |
| Φ | 0·00 | 28·97802 | 29·99812 |

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* The distance of the point at which the ray intersects the axis from the last surface, along the axis.

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These focal lengths are taken from another triple similar to the first, but having a ratio of aperture to focus of 10. They show that with spherical surfaces even a triple cannot be perfectly aplanatic.

Some focal lengths similarly traced of the 12·5 aperture ratio triple follow :—

| Ray. | Height from Vertex. | Back Focus. | Focus. |
|---|---|---|---|
| A′ | 1·50 | 36·83056 | 37·49001 |
| B′ | ” | 36·83988 | 37·50140 |
| C | ” | 36·84683 | 37·50739 |
| Λ | 0·00 | 36·84544 | 37·50000 |
| ” | 1·50 | 36·84157 | 37·50349 |
| F | ” | 36·83996 | 37·50354 |
| Φ | ” | 36·84110 | 37·50481 |
| G′ | ” | 36·85882 | 37·52463 |

*Fig. 1.*

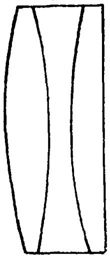
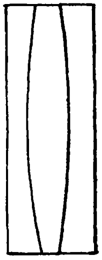


*Fig. 2.*



*Note.*—In order to construct a triple of the form shown in fig. 1 it is necessary to use two flints and a crown. The Steinheil type, fig. 2, is constructed by employing two crowns and a flint. One of these should be a heavy barium crown. The curves are steeper.

By using rays A′, Λ, and Φ as the basis of the colour correction, instead of B′, Λ, and Φ, the radii may be still further lengthened.

### *Data of Trigonometrical Trace.*

For axial ray $H_0 = F$, for edge ray $H =$ height from vertex. Then for parallel incident ray, $R_0$ is arbitrary.

$$R_2 = \frac{(H_0 - t\delta_0)\mu - 1}{-\delta_2 + \delta_0(\mu - 1)}, \; \delta_2 = \beta_{0,2} - \delta_0, \; \beta_{0,2} = \frac{1}{f}^{*}. \; R_4 = R_2$$

(or arbitrary).

$$R_6 = \frac{(A'_4 - t_5)\beta_{0,4}}{\theta_6 + \beta_{0,6}}, \; \theta_6 = \frac{\delta_6}{1 - \mu'}, \; \delta_6 = \beta_{4,6} - \delta_4, \; \beta_{4,6} = \frac{1}{f}^{*}. \; R_8 = R_6$$

(or arbitrary).

$$R_{10} = \frac{(A'_8 - t_8)\beta_{0,8}}{\theta_{10} + \beta_{0,8}}, \; \theta_{10} = \frac{\delta_{10}}{1 - \mu''}, \; \delta_{10} = \beta_{8,10} - \delta_8 \, . \; \beta_{8,10} = \frac{1}{f''}^{*}.$$

$t_1 = a + \beta$, $a = R_0 - \sqrt{R_0^2 - d^2}$, $\beta = R_2 - \sqrt{R_2^2 - d^2}$. $d =$ semi-aperture.

$t_5$ and $t_9$ are found similarly. $t_3$ and $t_7$, air thickness, are arbitrary.

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[*] $\beta_{0,2}$ $\beta_{4,6}$ and $\beta_{8,10}$ are only approximately equal to $\frac{1}{f}$, $\frac{1}{f'}$ and $\frac{1}{f''}$; an adjustment must be made for thickness. This must be ascertained by the trigonometrical trace. The values in the example given were less by something near unity in the first decimal place.

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**Axial Ray.**

$$\theta_0 = \frac{H_0}{R_0}$$

$$\phi_0 = \frac{\theta_0}{\mu}$$

$$\delta_0 = \theta_0 - \phi_0$$

$$A'_0 - R_0 = \frac{R_0 \phi_0}{\delta_0}$$

$$\theta_2 = \frac{(A'_3 - R_2 - t_1)\delta_0}{R_2}$$

$$\phi_2 = \theta_2 \mu$$

$$\delta_2 = \theta_2 - \phi_2$$

$$\beta_{0,2} = \delta_0 + \delta_2$$

$$A'_2 - R_2 = \frac{R_2 \phi_2}{\beta_{0,2}}$$

$$\theta_4 = \frac{(A'_2 - R_4 - t_3)\beta_{0,2}}{R_4}$$

$$\phi_4 = \frac{\theta_4}{\mu}$$

$$\delta_4 = \theta_4 - \phi_4$$

$$\beta_{0,4} = \delta_4 + \beta_{0,2}$$

$$A'_4 - R_4 = \frac{R_4 \phi_4}{\beta_{0,4}}$$

$$\theta_6 = \frac{(A'_4 - t_5 - R_6)\beta_{0,4}}{R_6}$$

$$\phi_6 = \theta_6 \mu'$$

$$\delta_6 = \theta_6 - \phi_6$$

$$\beta_{0,6} = \delta_6 + \beta_{0,4}$$

$$A'_6 - R_6 = \frac{R_6 \phi_6}{\beta_{0,6}}$$

$$\theta_8 = \frac{(A'_6 - R_8 - t_9)\beta_{0,6}}{R_8}$$

$$\phi_8 = \frac{\theta_8}{\mu''}$$

$$\delta_8 = \theta_8 - \phi_8$$

$$\beta_{0,8} = \delta_8 + \beta_{0,6}$$

$$A'_8 - R_8 = \frac{R_8 \phi_8}{\beta_{0,8}}$$

$$\theta_{10} = \frac{(A'_8 - R_{10} - t_9)\beta_{0,8}}{R_{10}}$$

$$\phi_{10} = \theta_{10} \mu''$$

$$\delta_{10} = \theta_{10} - \phi_{10}$$

$$\beta_{0,10} = \delta_{10} + \beta_{0,8}$$

$$A'_{10} - R_{10} = \frac{R_{10} \phi_{10}}{\beta_{0,10}}$$

$$A'_{10} = (A'_{10} - R_{10}) + R_{10}$$

$$F_{0,10} = \frac{H_0}{\beta_{0,10}}$$

**Edge Ray.**

$$\sin \theta_0 = \frac{H}{R_0}$$

$$\sin \phi_0 = \frac{\sin \theta_0}{\mu}$$

$$\delta_0 = \theta_0 - \phi_0$$

$$A'_0 - R_0 = \frac{R_0 \sin \phi_0}{\sin \delta_0}$$

$$\sin \theta_2 = \frac{(A'_0 - R_2 - t_1) \sin \delta_0}{R_2}$$

$$\sin \phi_2 = \mu \sin \theta_2$$

$$\delta_2 = \theta_2 - \phi_2$$

$$\beta_{0,2} = \delta_0 + \delta_2$$

$$A'_2 - R_2 = \frac{R_2 \sin \phi_2}{\sin \beta_{0,2}}$$

etc., etc., sines of angles being substituted for angles where shown.

**Notes.**

$A'_{10}$ is the distance of the last surface, along the axis, from the point at which the emergent ray intersects the axis.

$F_{0,10}$ is the distance of the optical centre of the triple, along the axis, from the point above named.

In the working for axial ray, $\beta_{0,10}$ = one second of arc in every case.

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# *On the Determination of the apparent Diameter of a fixed Star.*

By Major P. A. MacMahon, D.Sc., F.R.S.

Some sort of an idea of the size of a fixed star may be formed from a knowledge of its parallax and of its magnitude, but astronomers have not felt justified in placing much reliance upon any estimate obtained in this manner. A comparison between the light of the Sun and that of a star is difficult because of the immense difference between them. Statements ascribing to $\alpha$ Centauri and Sirius intrinsic splendours equal to 2·3 and 63 times that of the Sun respectively can only be regarded as indications that the truth is probably somewhere in the neighbourhood of these numbers.

Whether these differences of splendour are due to differences of diameter or of the amount of light emitted by equal areas requires further observation, and a direct method of determining a star's apparent diameter is much to be desired. For this purpose those stars which periodically experience occultation by the Moon are the most favourably situated for investigation. It will be remembered that certain stars were first shown to be double by peculiarities observed during occultation, and the idea comes into the mind that it may be possible to subject the phenomenon of lunar occultation to a refined and expanded photographic examination by employment of the long, rapidly moving films which are used in the modern bioscope.

The time taken by the Moon to occult a fixed star has been described as "instantaneous," or "less than the twinkling of an eye," and in other terms also indicating the opinion that the time in question is at once inappreciable and immeasurable. Very minute intervals of time are, however, measurable. Sir Andrew Noble's chronograph measures $\frac{1}{1000000}$ of a second. Professor Worthington has recently photographed the splash formed by a drop of water in a time estimated to be $\frac{3}{1000000}$ second, which does not differ much from a light-kilometre.

The question then arises as to the time probably taken by the Moon to occult an average fairly bright star.

I had made a few calculations based upon parallaxes which astronomers regard as being reliable within a small probable error, and had found that the time might be $\frac{1}{1000}^{\mathrm{s}}$, when I took the opportunity of consulting Professor Dyson. He assumed the star to have a parallax $0''\!\cdot\!10$, and to be of the same actual diameter as the Sun; this leads to an apparent diameter $\frac{1}{1000}''$, and the time taken by the Moon to effect a direct occultation would then be about $\frac{1}{500}^{\mathrm{s}}$.

In a letter to me, dated 1908 November 1, Professor Dyson proceeds: "Thus exposures of $\frac{1}{1000}^{\mathrm{s}}$ or rather less would be required. With the Astrographic (13 in. O.G.) telescope a star of $9^{\mathrm{m}}\!\cdot\!0$ registers so as to give a measurable image in $10^{\mathrm{s}}$. A star of